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Semi-classical probe strings on giant gravitons backgrounds

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Abstract: In the first part of this paper we study two $\mathbb{Z}_2$ symmetries of the LLM metric, both of which exchange black and white regions. One of them which can be interpreted as the particle-hole symmetry is the symmetry of the whole supergravity solution while the second one is just the symmetry of the metric and changes the sign of the fivefrom flux. In the second part of the paper we use closed string probes and their semi-classical analysis to compare the two 1/2 BPS deformations of $AdS_5 \times S^5$, the smooth LLM geometry which contains localized giant gravitons and the superstar case which is a solution with naked singularity corresponding to smeared giants. We discuss the realization of the $\mathbb{Z}_2$ symmetry in the semi-classical closed string probes point of view.

Keywords: AdS-CFT and dS-CFT Correspondence, Penrose limit and pp-wave background.

Contents

1. Introduction

The AdS/CFT correspondence (duality) [1], as the only explicit example in which the holographic formulation of quantum gravity (string theory) is realized, has been the corner stone of the string theory studies in the past seven years. The above correspondence is a strong/weak duality in the sense that when the gauge theory is perturbative the string theory sigma model is strongly coupled and vice-versa. Hence usually one can only perform perturbative computations in one side of the duality. Recently, however, it has been shown that there is a certain large quantum numbers limit, the BMN sector [2] and the "semi-classical" strings [3, 4], in which the gauge theory and the string theory sides are both perturbatively accessible, for a review e.g. see [5, 6].

Besides the large quantum numbers limit, noting the large supersymmetry of either sides of the duality, namely $\mathrm{PSU}(2,2|4)$ superalgebra, one can focus on the BPS information which are protected by supersymmetry. One might perform computations with

BPS objects in a weakly coupled regime on either sides of the duality and due to supersymmetry expect the same computation to be still valid at strong coupling. The $\mathrm{PSU}(2,2|4)$ is a superalgebra with 32 supercharges and hence we have the option of looking at various BPS sectors. The (dynamically) simplest BPS sector is the 1/2 BPS one. In the $\mathcal{N}=4$ $\mathrm{U}(N)$ gauge theory side the highest weight state of the 1/2 BPS multiplet is a chiral primary operator. These operators are only a function of one of the three complex scalars present in the $\mathcal{N}=4$ vector multiplet. Let us denote this scalar, which is an $N \times N$ matrix, by Z. The 1/2 BPS sector, i.e. the chiral primary operators, are then all possible gauge invariant combinations of various powers of Z (and not $Z^\dagger$). Since we are dealing with $N \times N$ matrices, Z^k for $k > N$ are not independent matrices. The scaling dimension, Δ, of 1/2 BPS operators is protected by supersymmetry and is equal to their classical (engineering) dimension which is also equal to their R-charge, J. All the n-point functions of these BPS operators are protected against gauge theory loop corrections, i.e. they are g_{YM} independent. This makes 1/2 BPS sector a perfect laboratory for studying the combinatorics and the $1/N$ expansion behavior.

Giving the R-charge J does not completely specify the chiral primary operator, e.g. the operator can be single, double or multi trace and in general a linear combination of multi-trace operators. To obtain a gauge invariant operator made out of product of J Z_{ij}'s we need a $\mathrm{U}(N)$ tensor to contract the $N \times N$ indices, explicitly $\mathcal{O}_T = T^{i_1 i_2 \cdots i_J}_{j_1 j_2 \cdots j_J}\, Z_{i_1 j_1} Z_{i_2 j_2} \cdots Z_{i_J j_J}$. Therefore, an operator is completely specified by giving $T^{i_1 i_2 \cdots i_J}_{j_1 j_2 \cdots j_J}$. This can be done through a Young tableau made out of J boxes.

It has been shown that the sector of the $\mathcal{N}=4$ $\mathrm{U}(N)$ SYM on $R \times S^3$ which is only made out of Z and $Z^\dagger$ that are constant on the S^3, is equivalent to an N fermion system in two dimensional harmonic oscillator potential [7−9]. Furthermore, the 1/2 BPS sector is equivalent to reducing the 2d fermion system to a one dimensional one. The phase space of the N fermion system can be described by the same Young tableau discussed above [7, 8].

In [10] Lin-Lunin-Maldacena (LLM) constructed the type IIB supergravity solutions which are dual to 1/2 BPS states of $\mathcal{N}=4$ supersymmetric Yang-Mills theory. As we will briefly review in section 2, these solutions are specified by a single function which essentially obeys a six dimensional Laplace equation, sourced on a two dimensional plane. This plane, via AdS/CFT, should be identified with the phase space of the above mentioned one dimensional fermion system.

The paper is organized as follows. In section 2 we will review LLM construction and discuss the discrete symmetries of the solutions. We will show that there are two $\mathbb{Z}_2$ symmetries of the LLM metrics, both of which exchange black and white regions. We study these $\mathbb{Z}_2$ symmetries from the supergravity, superalgebra and the $\mathcal{N}=4$ SYM viewpoints. In section 3 we consider the deformation of AdS solution by adding giant gravitons. There are two ways to do that, one leads to a singular and the other to smooth geometries. We will probe these geometries by closed spinning strings and discuss how the two smooth and singular 1/2 BPS geometries are viewed by the closed string probes. The last section 4 is devoted to concluding remarks.

2. $\mathbb{Z}_2$ symmetries of LLM geometries

The AdS/CFT correspondence implies that the type IIB supergravity solutions corresponding to the chiral primaries of the $\mathcal{N}=4$ SYM should have the following properties:

(i) The scaling dimension of the 1/2 BPS operators is protected by supersymmetry and hence the corresponding SUGRA solution should be a static solution with a globally defined time-like Killing vector field.

(ii) The chiral primary operators are invariant under $\mathrm{SO}(4) \subset \mathrm{SU}(4)_R$ as well as $\mathrm{SO}(4) \subset \mathrm{SO}(4,2)$, therefore the SUGRA solution should have an $\mathrm{SO}(4) \times \mathrm{SO}(4)$ isometry.

This class of solutions does not have non-compact isometries other than the non-compact U(1) factor associated with the translation along the time-like Killing vector. The supersymmetry of the system in ensured by checking the Killing spinor equations and that they have 16 independent solutions. In fact LLM constructed their solutions using the Killing spinors and spinor bi-linear techniques developed in [11].

The LLM solutions are all deformations[1] of the two maximally supersymmetry solutions of type IIB backgrounds, namely $AdS_5 \times S^5$ and the plane-wave [5].[2] The supersymmetry of the LLM geometries are then a subgroup of the AdS or plane-wave superalgebras; specifically that is $\mathrm{PSU}(2|2) \times \mathrm{PSU}(2|2) \times \mathrm{U}(1)$ [5, 13], where the U(1) factor corresponds to the time translations. In terms of $\mathcal{N}=4$ SYM the eigenvalues of this U(1) correspond to the scaling dimension of the operators.

Being a 1/2 BPS solution with the required supersymmetry, the dilatino variation for the LLM geometries must be fulfilled identically. This implies that the 1/2 BPS solutions should have a constant dilaton and the NSNS twoform should vanish. The gravitino variation and the 1/2 BPS condition restricts the solution further to have zero axion and RR twoform and only the selfdual fiveform field can be turned on. The LLM geometries are given by [10]

$$\begin{aligned} ds^2 &= -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2 \,, \\ h^{-2} &= 2y \cosh G \,, \qquad z = \frac{1}{2} \tanh G \,, \\ y\partial_y V_i &= \epsilon_{ij} \partial_j z \,, \qquad y(\partial_i V_j - \partial_j V_i) = 2\epsilon_{ij}\partial_y z \,, \qquad i,j = 1,2 \,. \end{aligned} \tag{2.1}$$

The self dual five-form field strength is also nonzero

$$F_5 = F \wedge d\Omega_3^2 + \tilde{F} \wedge d\tilde{\Omega}_3^2 \,, \tag{2.2}$$

[1] Classically and just by supergravity considerations these are continuous deformations of AdS or the plane-wave geometries. Considering the semi-classical or quantum arguments, these deformations are, however, parameterized by a discrete parameter [10]. This point will be discussed further later on.

[2] It is worth noting that LLM solutions are not connected to the third and the only remaining maximally supersymmetric type IIB background [12], the flat space.

where

$$
\begin{aligned}
F &= -\frac{1}{4}\left[d\left(y^2 e^{2G}(dt+V)\right) + y^3 *_3 d\left(\frac{z+\frac{1}{2}}{y^2}\right)\right]\\
\tilde{F} &= -\frac{1}{4}\left[d\left(y^2 e^{-2G}(dt+V)\right) + y^3 *_3 d\left(\frac{z-\frac{1}{2}}{y^2}\right)\right].
\end{aligned}
\tag{2.3}
$$

To fix our notation we parameterize the two spheres as follows

$$
\begin{aligned}
d\Omega_3^2 &= d\theta_1^2 + \cos^2\theta_1(d\theta_2^2 + \cos^2\theta_2 d\theta^2)\\
d\tilde{\Omega}_3^2 &= d\psi_1^2 + \cos^2\psi_1(d\psi_2^2 + \cos^2\psi_2 d\psi^2)\,.
\end{aligned}
\tag{2.4}
$$

As we see from the above expressions, the 1/2 BPS condition is very restrictive and the whole solution is completely specified through a single function z which satisfies the following equation

$$
\partial_i\partial_i z + y\partial_y\left(\frac{\partial_y z}{y}\right) = 0\,. \tag{2.5}
$$

The above equation is a six dimensional Laplace equation for $\Phi = \frac{1}{y^2}z$. Therefore, (2.5) can be easily solved once we specify the (delta function type) sources on the right hand side of the equation. These sources should be placed on the $y=0$ plane which is a two dimensional plane parameterized by (x_1,x_2), explicitly, they are of the form $\frac{1}{y}\rho(x_1,x_2)\delta(y)$.

It is reasonable to assume that only non-singular supergravity solutions should be dual to 1/2 BPS chiral primary operators of the gauge theory. The smoothness condition implies that z must obey the boundary condition $z=\pm\frac{1}{2}$ at $y=0$ on the (x_1,x_2) 2-plane. Let us assign black and white colors to the $z=-\frac{1}{2}$ and $z=\frac{1}{2}$ regions of the $y=0$ plane, respectively. This is in perfect agreement with what we expect from the 1/2 BPS operators in the dual SYM theory: in the 2d fermion language, the black and white regions directly correspond to fermions (black) and holes (whites) [8, 10]. From this observation, however, one learns an important fact about the (x_1,x_2) plane which is not coming out of the classical supergravity considerations alone:[3] In the quantized gravity theory the (x_1,x_2) plane is a noncommutative plane with[4]

$$
[x_1,x_2] = 2\pi i\ l_{Pl}^4\,. \tag{2.6}
$$

(Note that in the conventions we are using, which is the same as the one adopted in [10], x_1,x_2 and y coordinates have dimension of length^2.) This in particular implies that there is a minimal area that one can probe on the (x_1,x_2) plane and that (2.6) defines an orientation on the (x_1,x_2) plane. The noncommutativity of the (x_1,x_2) plane is necessary to ensure that we are not going to get solutions with naked singularities (superstars [16]) as a limit of the smooth LLM geometries.

[3]The LLM solutions are completely described by a single function z. One may try using the mini-superspace quantization of the LLM geometries. This process has been carried out in [14] and shown that it, in fact, reproduces (2.6).

[4]According to the tiny graviton matrix theory conjecture [13] in the fully quantized gravity/string theory, not only the (x_1,x_2) plane but also the the spheres S^3 and $\tilde{S}^3$ are also "quantized" fuzzy spheres [13, 15].

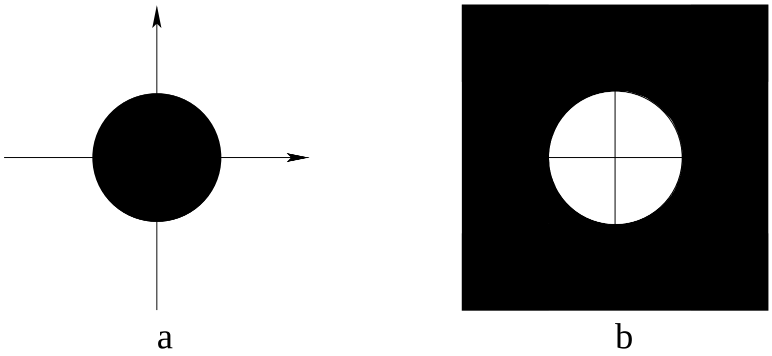

Figure 1: The configurations which correspond to $AdS_5\times S^5$.

Eq. (2.5) is a linear equation and hence the most general solutions can be constructed as linear combinations of simpler solutions, provided that the sum is also respecting the boundary conditions. In other words, in general a solution is specified by a configuration of black droplets in the (x_1, x_2) plane. For further studies in this direction see e.g. [17, 18].

The simplest example is a black disc centered at the origin, figure 1a. Setting $\tilde{z} = z - \frac{1}{2}$ the solution in given by

$$\begin{aligned}
\tilde{z} &= \frac{1}{2}\,\frac{r^2 - r_0^2 + y^2}{\sqrt{(r^2+r_0^2+y^2)^2 - 4r^2r_0^2}} - \frac{1}{2}\,,\\
V_\phi &= -\frac{1}{2}\,\frac{r^2 + r_0^2 + y^2}{2\sqrt{(r^2+r_0^2+y^2)^2 - 4r^2r_0^2}} + \frac{1}{2}\,, \qquad V_r = 0\,. \qquad (2.7)
\end{aligned}$$

Here we have chosen the polar coordinates r, ϕ in the (x_1, x_2) plane. In terms of $\tilde{z}$ the boundary condition is given by $\tilde{z} = -1$ for black region of radius r_0 and $\tilde{z} = 0$ for white region and the parameter G is given by

$$e^G = \sqrt{\frac{1+\tilde{z}}{-\tilde{z}}}\,. \qquad (2.8)$$

Now let us discuss two $\mathbb{Z}_2$ symmetries of the LLM metrics, both of which exchange black and white regions. One such example has been depicted in figure 1. The first $\mathbb{Z}_2$ symmetry we would like to discuss is the one which may be interpreted as the particle-hole symmetry in the corresponding two dimensional fermion system [8, 19]. This $\mathbb{Z}_2$ can be discussed at supergravity level and the LLM solutions, the superalgebra level and also in the dual gauge theory setup and on the chiral primary operators. This means that the $\mathbb{Z}_2$ symmetry is a non-anomalous symmetry and remains at quantum (gravity) level.

2.1 $\mathbb{Z}_2$ symmetry at the supergravity level

Consider the $\mathbb{Z}_2$ transformation generated by

$$G \longleftrightarrow -G \qquad (2.9a)$$

$$(x_1, x_2) \longleftrightarrow (x_2, x_1)\,. \qquad (2.9b)$$

Under (2.9a) it is easily seen that $z \longleftrightarrow -z$ which is equivalent to black$\longleftrightarrow$ white regions on the (x_1, x_2) plane. Noting (2.1), it is easily seen that $V_i dx^i$ is invariant under the above transformation. Therefore, not only the metric, but the whole geometry is invariant under the above $\mathbb{Z}_2$ transformation.

Eq. (2.9a) implies that under the above symmetry we are exchanging the two three spheres in (2.1) and hence it is exchanging the two SO(4) factors of the isometries. The (2.9b) is nothing but the parity on the (x_1, x_2) plane. This changes the orientation of the (x_1, x_2) plane and hence is not compatible with (2.6).

There is another $\mathbb{Z}_2$ transformation:

$$G \longleftrightarrow -G \tag{2.10a}$$

$$t \longleftrightarrow -t\,, \tag{2.10b}$$

under which $V_i dx^i \longleftrightarrow -V_i dx^i$, which is a symmetry of the background metric (2.1) and also preserves (2.6). This $\mathbb{Z}_2$, however, is not a symmetry of the whole solution, because under (2.10) the fiveform flux changes sign. Transformation (2.10), and in particular (2.10a), is again changing the black and white regions while keeping the orientation on the (x_1, x_2) plane. Therefore, there is a twofold degeneracy in the LLM solutions for a given configuration of black and white regions on the (x_1, x_2) plane. These two solutions differ in the orientation on the (x_1, x_2) plane and/or the direction of the time coordinate. Although the both result in the same metric, the corresponding to fiveform fluxes differ in a sign.

2.2 $\mathbb{Z}_2$ at the level of SUSY algebra

The LLM geometries are 1/2 BPS and hence it is interesting to analyze the action of the above $\mathbb{Z}_2$'s on the supersymmetry algebra. Before that, however, we need to introduce some fermionic notation. The $AdS_5 \times S^5$ geometry has PSU(2, 2|4) supersymmetry which is an algebra with 32 (real) fermionic generators. The supercharges of this algebra fall into the fermionic representations of the $so(4,2) \simeq su(2,2)$ and $so(6) \simeq su(4)$ algebras. Explicitly, the supercharges can be labeled as $Q_{I\hat{J}}$ (and its complex conjugate) where $\hat{I}, J$ are Weyl indices of $su(2,2)$ and $su(4)$, respectively. It is worth noting that for a given $AdS_5 \times S^5$ geometry we have freedom to choose the sign of the corresponding fiveform flux over the S^5. These two $AdS_5 \times S^5$ geometries, although both are maximally supersymmetric, preserve two different PSU(2, 2|4) superalgebras. The supercharges of these two differ in the chirality of the $su(2,2)$ and $su(4)$ fermions, i.e. if one of them have supercharges of the form $Q_{\hat{I}J}$, the other one has $Q_{\dot{\hat{I}}\dot{J}}$ supercharges (cf. [5, appendix D]). These two $AdS_5 \times S^5$ spaces come as near horizon limit of N D3-branes or anti D3-branes. Next, consider a Weyl fermion of $su(4)$ or $su(2,2)$, ψ_I. This fermion can be decomposed under $so(4) \times u(1) \subset su(4)$ or $su(2,2)$, as $(\psi^+_\alpha,\ \psi^-_{\dot{\alpha}})$, where $\alpha,\ \dot{\alpha} = 1, 2$ are the standard four dimensional Weyl indices and the $+$ and $-$ correspond to the $u(1)$ charge. (The $\psi_{\dot{I}}$ would then decompose into $(\psi^-_\alpha,\ \psi^+_{\dot{\alpha}})$.) For more detailed discussion on similar fermionic notation see [5, appendix D].

The supersymmetry of the LLM solutions is then a subalgebra of either of the two PSU(2, 2|4) superalgebras discussed above. The supercharges $Q_{\hat{I}J}$ (or $Q_{\dot{\hat{I}}\dot{J}}$) under $\mathrm{SO}(4) \times \mathrm{SO}(4) \times \mathrm{U}(1)_t \times \mathrm{U}(1)$ decompose into four fermions which differ in the relative sign of the U(1) charges. Physically the $\mathrm{U}(1)_t$ corresponds to the translation along the time-like Killing direction and the charge of the other U(1) is correlated with the orientation on the (x_1, x_2) plane.

The supercharges of the PSU(2, 2|4), $Q_{I\hat{J}}$, decompose into $(q^{++}_{\alpha\beta},\ q^{--}_{\dot\alpha\dot\beta})$ and $(q^{+-}_{\alpha\dot\beta},\ q^{-+}_{\dot\alpha\beta})$, each set giving rise to a PSU(2|2) × PSU(2|2) × U(1) algebra, which is the supersymmetry of the LLM geometries. Under $G \longleftrightarrow -G$, the α and β indices are exchanged. Change of orientation on (x_1, x_2) plane (while keeping the six dimensional chirality) implies that $\psi^+_\beta \longleftrightarrow \psi^-_{\dot\beta}$. Altogether the $\mathbb{Z}_2$ symmetry (2.9) on the fermionic indices act as

$$\psi^{+-}_{\alpha\dot\beta} \longleftrightarrow \psi^{+-}_{\beta\dot\alpha}\,, \qquad \psi^{--}_{\dot\alpha\dot\beta} \longleftrightarrow \psi^{++}_{\beta\alpha}\,. \tag{2.11}$$

At the level of the PSU(2|2) × PSU(2|2) × U(1) superalgebra the $\mathbb{Z}_2$ exchanges either the two PSU(2|2) factors or the two SU(2)'s inside each of PSU(2|2)'s. This is exactly the same as the $\mathbb{Z}_2$ symmetry of the plane-wave background, for an extensive discussion on this see [5].

Therefore, if we start with a PSU(2|2) × PSU(2|2) × U(1) with supercharges of the form $(q^{++}_{\alpha\beta},\ q^{--}_{\dot\alpha\dot\beta})$ we obtain the same PSU(2|2) × PSU(2|2) × U(1) superalgebra where the two PSU(2|2) factors are exchanged. In other words, the $\mathbb{Z}_2$ (2.9) keeps the supergravity solution and the form of the corresponding superalgebra invariant; similarly if we choose to work with supercharges of the form $(q^{+-}_{\alpha\dot\beta},\ q^{-+}_{\dot\alpha\beta})$.

The second $\mathbb{Z}_2$, (2.10), changes the fermions as

$$\psi^{++}_{\alpha\beta} \longleftrightarrow \psi^{-+}_{\beta\dot\alpha}\,, \qquad \psi^{--}_{\dot\alpha\dot\beta} \longleftrightarrow \psi^{+-}_{\dot\beta\alpha}\,, \tag{2.12}$$

and hence the PSU(2|2) × PSU(2|2) × U(1) superalgebra is not invariant under this $\mathbb{Z}_2$, but it goes over to another PSU(2|2) × PSU(2|2) × U(1) which is a subgroup of a PSU(2, 2|4) algebra whose supercharges are of the form $(q^{--}_{\alpha\beta},\ q^{++}_{\dot\alpha\dot\beta})$ and $(q^{+-}_{\dot\alpha\beta},\ q^{-+}_{\alpha\dot\beta})$. This PSU(2|2) × PSU(2|2) × U(1) is a subgroup of the PSU(2, 2|4) superalgebra whose generators are in the $Q_{\hat{I}\hat{J}}$ representation.

2.3 $\mathbb{Z}_2$ at the level of $\mathcal{N} = 4$ SYM

The LLM geometries are, via AdS/CFT, dual to deformations of the $\mathcal{N} = 4$ U(N) SYM by 1/2 BPS chiral primary operators. These operators can conveniently be described by U(N) Young tableaux [7, 15, 17] which are in turn in one-to-one correspondence with the black and white rings on the (x_1, x_2) plane [10, 19].

One may now ask how does the $\mathbb{Z}_2$ symmetries act on the set of chiral primary operators or Young tableaux. From the $\mathcal{N} = 4$ SYM viewpoint, there are two kinds of 1/2 BPS chiral primary operators, they either have $\Delta = J$ or $\Delta = -J$, where Δ and J are scaling dimension and R-charge respectively. The orientation in the (x_1, x_2) plane is determining the sign of $\Delta - J$. (To see this note [10, equations (2.24)–(2.26)] and recall that $J = -i\frac{\partial}{\partial\phi}$ and $\Delta = i\frac{\partial}{\partial t}$.) Under the (2.9b) $\mathbb{Z}_2$ then $\Delta - J \leftrightarrow -(\Delta - J)$ while keeping Δ fixed. As we see if we choose to work with chiral primaries made only out of Z, they all have $\Delta - J = 0$ and hence invariant under the orientation change in (x_1, x_2) plane. Equation (2.9a), however,

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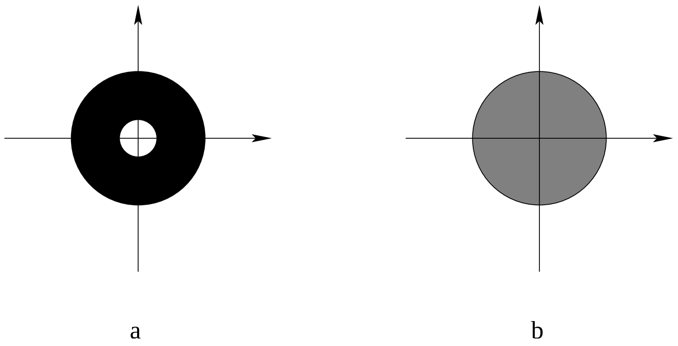

Figure 2: *AdS* deformation by adding giant gravitons. The deformed solution can lead either to a smooth geometry (a) or a geometry with naked singularity (b). In (a), r_1 is the radius of the outer circle and r_2 the radius of the inner white circle.

affects how the U(N) indices of the gauge theory operators are contracted, i.e. it is reflected in the Young tableaux. At the level of the Young tableaux exchanging black and white corresponds to exchanging the vertical and horizontal axis of the tableau, or equivalently it is the inversion under the line at 45° on the Young diagram. To see this recall the black and white assignment given a Young tableau; the latter maybe found in [19].[5]

The second $\mathbb{Z}_2$ at the level of the $\mathcal{N}=4$ SYM acts like a time reversal, which despite of being a symmetry of the action changes the chirality and representation of fermions under the R-symmetry, essentially as given in (2.12).

3. Deformation of AdS with giant gravitons

We start with the $AdS_5 \times S^5$ solution which in the LLM construction is given by a black disk in the (x_1, x_2) plane and add spherical D3-branes (giant gravitons) wrapped on $\tilde{S}^3 \in S^5$. Using the LLM setup we can compute the back reaction of the giant gravitons on the geometry. In this viewpoint the corresponding solution is described by a black disk with a small white hole in it. The white hole in the middle of the black disk represents a collection of smeared giant gravitons with maximum size. The number of giant gravitons are fixed by the radius of the hole (see figure 2a). When the radius of the white hole is small the good description is given in terms of the giant gravitons, while for the large radius the better description may be given in terms of the smeared dual giant gravitons (giants wrapping the S^3 inside AdS_5). The corresponding supergravity solution is given by (2.1) with

$$2\tilde{z} = \frac{r^2 - r_1^2 + y^2}{\sqrt{(r^2 + r_1^2 + y^2)^2 - 4r^2 r_1^2}} - \frac{r^2 - r_2^2 + y^2}{\sqrt{(r^2 + r_2^2 + y^2)^2 - 4r^2 r_2^2}} \tag{3.1}$$

[5] In a recent paper, [20], a "classical" limit of a Young tableau has been discussed. In this classical limit the edge of a Young tableau which is a stairs-like line is replaced (or approximated) with a one dimensional curve $y(x)$ where x is the radial direction in the (x_1, x_2) plane. In their notation $z = \frac{1}{2}\frac{y'-1}{y'+1}$ (cf. [20, equation (111)]). The $\mathbb{Z}_2$ (2.9) corresponds to taking $y' \to 1/y'$. That is, the $\mathbb{Z}_2$ dual Young tableau is a curve whose tangent at each point makes the same angle with $y = x$ line as the original curve.

We note that the obtained gravity is smooth without any singularity and horizon, though adding giant gravitons might also lead to a singular geometry. In fact the singular background representing smeared giant gravitons has already been studied. These are the solutions called superstars [16]. The corresponding solution in the LLM notation is given by (2.1) with

$$\begin{aligned} \tilde{z} &= \frac{\rho}{2}\left(\frac{r^2 - r_0^2 + y^2}{\sqrt{(r^2 + r_0^2 + y^2)^2 - 4r^2 r_0^2}} - 1\right) \\ V_\phi &= \frac{-\rho}{2}\left(\frac{r^2 + r_0^2 + y^2}{\sqrt{(r^2 + r_0^2 + y^2)^2 - 4r^2 r_0^2}} - 1\right). \end{aligned} \tag{3.2}$$

It is straightforward to check that the above $\tilde{z}$ satisfies the six dimensional Laplace equation (2.5) with the following source at $y = 0$ (on the (x_1, x_2) plane):

$$\begin{aligned} \tilde{z} &= -\rho\,, \qquad & 0 \le r < r_0\,, \\ \tilde{z} &= 0 \qquad & r \ge r_0\,. \end{aligned} \tag{3.3}$$

As we can see $\tilde{z}$ now takes values other than $0, -1$ and hence the solution is singular. For the superstar solutions [16], $-1 < \tilde{z} < 0$ and it has been argued that LLM solutions with $-1 < \tilde{z} < 0$ have generically naked, null singularities [19, 21, 22].[6]

The $\tilde{z}$ of (3.3) can be conveniently denoted by an extension of the LLM color-coding on the (x_1, x_2) plane: Assign black to the region with $\tilde{z} = -1$, white to $\tilde{z} = 0$ and *gray* to $-1 < \tilde{z} < 0$ (this color-coding has been employed in [19, 20, 23]). In this coding the closer $\tilde{z}$ to -1, the darker the gray color and for larger $\tilde{z}$ the gray is brighter. In this color-coding the above superstar solution is a gray droplet of radius r_0 located at the center (see figure 2b).[7]

The black $\leftrightarrow$ white $\mathbb{Z}_2$ symmetry can be extended to the gray color-coding noting that under the $\mathbb{Z}_2$ symmetry $\tilde{z} \leftrightarrow -1 - \tilde{z}$ (together with a change in the orientation of the (x_1, x_2) plane), dark gray is replaced by a faint gray. In the case of the superstar the $\mathbb{Z}_2$ symmetry then takes $\rho \leftrightarrow 1 - \rho$. The $\tilde{z} = -1/2$ is the self-dual point.

[6]The values of $\tilde{z} > 0$ or $\tilde{z} < -1$ leads to spacetimes with closed time-like curve (CTC) pathologies [19, 21, 22]. As was discussed in [7, 17, 19] the dynamics of the half BPS sector of the $\mathcal{N} = 4$ SYM or the LLM geometries is conveniently captured by a $2d$ quantum Hall system, or equivalently by a $3d$ (noncommutative) Matrix Chern-Simons theory. In terms of this effective field theory, $\tilde{z} > 0$ or $\tilde{z} < -1$ corresponds to a Chern-Simons theory with level less than one which has problems with unitarity [19]. In other words existence of the closed time like curves is mapped into the non-unitarity of the effective field theory description. Here we do not consider these pathological cases.

[7]One may wonder if gray disk can be obtained as a limit of black and white rings. This is of course the case. For example consider a collection of L successive black and white rings of area b and w respectively. Then take the $L \to \infty$ limit while keeping bL and wL fixed. This is only possible if we relax the quatization of b and w. This will lead to a superstar solution with $\rho = b/w$. This is a simple example of the fact that a singular solution may arise as limit of smooth a geometry. As mentioned, for this to happen we should relax the "quantization" condition on the area in the (x_1, x_2) plane. In this viewpoint, the singularity of the superstar geometry is removed (or "resolved") by the *quantum effects*. We would like to thank Jorge Russo for discussion on this ponit.

JHEP11(2005)005

In sum, starting from AdS with N units of the fiveform flux and deforming it with giant gravitons with a given angular momentum $\mathcal{J}$, we may obtain smooth (or singular) geometries depending on whether giants are "localized" (or "smeared"). For the above giant graviton configuration

$$\text{Giant graviton}: \begin{cases} N = \dfrac{1}{4\pi l_p^4}(r_1^2 - r_2^2) \\ \mathcal{J} = \dfrac{r_2^2}{4\pi l_p^4} N \end{cases} \tag{3.4}$$

while for the superstar case

$$\text{Superstar}: \begin{cases} N = \dfrac{1}{4\pi l_p^4}\rho r_0^2 \\ \mathcal{J} = \dfrac{1}{8\pi l_p^4}(1-\rho) r_0^2 \ N. \end{cases} \tag{3.5}$$

Considering a single giant as an object with angular momentum N, one may define number of giants n as $n = \frac{\mathcal{J}}{N}$ [16]. For the case of the giant graviton of figure 2a, $n = \frac{1}{4\pi l_p^4} r_2^2$ is (half) of the area of the inner white region and for the superstar case of figure 2b $n = \frac{1}{2}\frac{1}{4\pi l_p^4}(1-\rho) r_0^2$, the extra factor of 1/2 in the superstar case is arising form the averaging and the fact that in this case the giants are smeared. For a fixed N, increasing $\mathcal{J}$ is then equivalent to adding more giants, in the localized case this is done by making the hole bigger while in the superstar case it means we are making the gray disk brighter.

To compare the effects of the deformation caused by the smeared and localized giant gravitons we use semiclassical closed strings probing the above two backgrounds with the same N and $\mathcal{J}$. We consider strings stretched along y direction and rotate along some angular directions on S^3 or/and $\tilde{S}^3$ and x_1, x_2 are set to zero. For this particular closed string probes the $\tilde{z}$ function is given by

$$\text{AdS}: \qquad \tilde{z} = -\frac{r_0^2}{y^2 + r_0^2} \tag{3.6a}$$

$$\text{Superstar}: \qquad \tilde{z} = -\frac{\rho r_0^2}{y^2 + r_0^2} \tag{3.6b}$$

$$\text{Giant}: \qquad \tilde{z} = -\frac{r_1^2}{y^2 + r_1^2} + \frac{r_2^2}{y^2 + r_2^2}\,. \tag{3.6c}$$

3.1 Closed string probes rotating in $\tilde{S}^3$

For simplicity let us first consider semiclassical closed string solutions which are stretched along y direction and rotate along one direction in $\tilde{S}^3$. The corresponding solution and conserved charges are given by

$$t = \kappa\tau\,, \qquad \psi = \nu\tau\,, \qquad y = y(\sigma) = y(\sigma + 2\pi) \tag{3.7}$$

$$E = \frac{\kappa}{2\pi\alpha'}\int_0^{2\pi} d\sigma\ 2y\cosh G\,, \qquad J = \frac{\nu}{2\pi\alpha'}\int_0^{2\pi} d\sigma\ y e^{-G}\,. \tag{3.8}$$

We would now like to find the dependence of E on the spin (or R-charge) J. Following [3] we consider the short and long string limits. From the Virasoro constraints we get

$$\text{AdS}: y'^2 + y^2(y^2+r_0^2)\left(\frac{\nu^2-\kappa^2}{y^2} - \frac{\kappa^2}{r_0^2}\right) = 0 \tag{3.9a}$$

$$\text{Superstar}: y'^2 + y^2(y^2+r_0^2)\left(\frac{\nu^2-\kappa^2}{y^2+(1-\rho)r_0^2} - \frac{\kappa^2}{\rho r_0^2}\right) = 0 \tag{3.9b}$$

$$\text{Giant}: y'^2 + y^2(y^2+r_1^2)(y^2+r_2^2)\left(\frac{\nu^2-\kappa^2}{y^4+2y^2r_2^2+r_1^2r_2^2} - \frac{\kappa^2}{(r_1^2-r_2^2)y^2}\right) = 0\,. \tag{3.9c}$$

In general the above can be thought of as a "zero energy" condition for a non-relativistic particle with a potential. Noting that the first term, the "kinetic" term, is positive definite and that we are looking for periodic solutions, the above can only be satisfied if the "potential" $V(y)$ is negative or zero and in the same locus its derivative is non-negative as well; that is, to have a periodic solution $V(y) \le 0$ and $V'(y) \ge 0$ should be satisfied simultaneously.

In the AdS case the potential has no minimum for any value of ν, κ and hence the periodicity condition can only be satisfied at the zeros of the potential and such zeros only exist for $\nu \ge \kappa$. At these zeros the derivative of the potential is always negative except for the $\nu = \kappa$ case where the zero of the potential is at $y = 0$. In this case we have a string shrunk to zero size and rotating along the ψ direction with the speed of light such that in the leading order we have $E = J$. The small fluctuations of this string around the solution (3.7) is actually probing the plane wave background [3].

3.1.1 The case of superstar

In the superstar case the situation is similarly to the AdS case, namely the potential is always negative with negative slope. Therefore, we won't get any closed string solution except for $y = 0$ where $V(y)$ and $V'(y)$ both vanish. This happens independently of values of ν and κ. In the particular case of $\nu = \kappa$, in the leading order, one finds

$$E = \frac{1}{\rho}\, J \tag{3.10}$$

in which the $\rho \to 1$ is a smooth limit that brings us back to the undeformed in the AdS case.

It is interesting to note that for $\frac{1}{\rho} = k \in \mathbb{Z}$, $E = kJ$ is in fact similarly to the case of BPS condition $E = J$ but the circle along which the particle is moving is now an S^1/Z_k orbifold. Explicitly, consider $AdS_5 \times S^5/Z_k$ orbifold and adopt the coordinate system in such a way that the orbifolding is acting on an $S^1 \in S^5$. For the untwisted sector of the orbifold $E = kJ$ (for example see [26]).

Eq. (3.10) becomes more interesting recalling the correspondence (equivalence) between the quantum Hall system (QHS) and the 1/2 BPS sector of the $\mathcal{N} = 4$ super Yang-Mills [17, 19], according which ρ is indeed the (average) density of the $2d$ fermions in the Landau levels (in units of the external magnetic field) [19]. In other words, ρ is equal to the filling factor. For the integer quantum Hall effect (IQHE) ρ is equal to one. For the fractional quantum

Hall effect (FQHE) which is described by Laughlin wavefunctions, however, inverse of the filling factor is quantized in integer steps and hence for this case $\rho = 1/k,\ k \in \mathbb{Z}$.[8] That is, a superstar system is equivalent to a fractional QHS which, as discussed above, is related to an orbifolded $\mathcal{N} = 4$ SYM, or a noncommutative Chern-Simons (NCCS) theory on $\mathbb{R}\times\mathbb{R}^2/Z_k$ orbifold. This is in line with the SYM/NCCS correspondence at levels not equal to identity proposed and discussed in [19]. This orbifold picture can be directly connected with the black/gray color-coding introduced earlier. Consider a black disk, perform the Z_k orbifolding and again redefine the angular coordinate to cover the $(0, 2\pi)$ region. In this redefinition the black region becomes $1/k$ times "fainter", becoming "gray".

One may wonder whether the Penrose limit of this geometry is what is seen by the small fluctuations around this zero size string solution. We will come back to this point later when we study the Penrose limit of the solutions.

3.1.2 The giant graviton case

In the giant graviton case still we do not get closed string solution for $\nu \le \kappa$ while for $\nu > \kappa$ there is a possibility to get semi-classical rotating closed string solution. The periodicity condition can be satisfied if

$$\frac{r_1}{r_2} \ge \frac{\nu^2+\kappa^2}{\nu^2-\kappa^2} \tag{3.11}$$

for which the potential takes positive values for $y_- \le y \le y_+$, where

$$y_\pm^2 + r_2^2 = \frac{(r_1^2-r_2^2)(\nu^2-\kappa^2)}{2\kappa^2}\left(1 \pm \sqrt{1-\frac{4\kappa^2\nu^2 r_2^2}{(r_1^2-r_2^2)(\nu^2-\kappa^2)^2}}\right). \tag{3.12}$$

As the potential has no minimum, the only acceptable solution is where the potential vanishes and has a positive slope at that point. That is at $y = y_-$. An interesting feature of this case is that the closed string cannot be longer than a maximum size given by $\sqrt{r_1 r_2}$ which corresponds to the length of string whose quantum numbers satisfy the equality in equation (3.11). (See figure 3).

In the $r_1 \gg r_2$ case one expects the effects of the giant graviton back reaction on the geometry to be small. To see this, note that in this limit

$$y_-^2 \simeq \frac{\kappa^2}{\nu^2-\kappa^2}\, r_2^2\,. \tag{3.13}$$

Expanding (3.9c) around y_-, in the leading order we obtain

$$y'^2 + y^2(y^2+r_2^2)\left(\frac{\kappa^2}{y_-^2} - \frac{\kappa^2}{y^2}\right) = 0 \tag{3.14}$$

[8] Using (3.5) it is readily seen that ρ in terms of characteristics of the background is given by

$$\rho = \frac{1}{1+\frac{2\mathcal{J}}{N^2}}$$

and hence integer $1/\rho$ happens when $\frac{2\mathcal{J}}{N^2} = \frac{2n}{N} \in \mathbb{Z}$. If $n/N \in \mathbb{Z}$ then $1/\rho$ is an *odd* integer, corresponding to a *fermionic* Laughlin wavefunction for which inverse of the filling factor is an odd number.

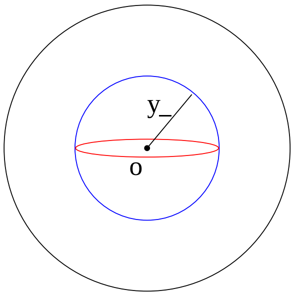

Figure 3: Semi-classical closed string solutions stretched along y direction with angular momentum in the $\tilde{S}^3$ in the giant graviton background.

which is equivalent to a semi-classical rotating closed string in the AdS background that rotates in the AdS space with the speed ν. Of course, noting the figures 1 and 2, this is expected from the $\mathbb{Z}_2$ symmetry we have studied in the previous section which exchanges the white and black regions.

We note, however, that the string can indeed distinguish this background from pure AdS through the next-to-leading order corrections. For example as we discussed the effects of such contributions lead to an upper limit on the length of the longest closed string one might have. In the leading order the turning point is given by (3.13) and in the long string limit where $\kappa \to \nu$ we can use the AdS approximation as long as $\frac{\kappa}{\nu} < 1 - \frac{r_2}{r_1}$ where the string probes an AdS background with radius r_2 which is produced by the giant gravitons and the energy is given by

$$E \approx J + \frac{r_2}{\pi\alpha'} \ln \frac{\alpha' J}{r_2}\,. \tag{3.15}$$

On the other hand in the limit $\frac{\kappa}{\nu} \to (1 - \frac{r_2}{r_1})$ or $y_- \to \sqrt{r_1 r_2}$ the above approximation breaks down and the string sees the whole geometry produced by the giant gravitons on the AdS background with the radius r_1.

In the short string limit where $\nu \gg \kappa$ one may also use the AdS approximation to find the behavior of the energy in terms of the angular momentum J. In this limit we get the Regge trajectory as in the flat space, as expected. In the short string limit the string shrinks to zero size as

$$y_- \approx \frac{r_2^2}{\sqrt{r_1^2 - r_2^2}} \frac{\kappa}{\nu} \tag{3.16}$$

and the prefactor in the Regge trajectory is changed as follows

$$E^2 \approx \frac{r_1^2}{\sqrt{r_1^2 - r_2^2}} \frac{J}{\alpha'}\,. \tag{3.17}$$

As we see, although in this case string still probes a flat space, the slope which is the effective string tension depends on the whole geometry including the giant gravitons effects.

As a conclusion we note that as long as these closed strings are concerned the background is very similar to AdS geometry, however, in the giant graviton case the folded string cannot be longer than $L = \sqrt{r_1 r_2}$. In the $r_1 \gg r_2$, $L^4 = nR^4$, where n is the number of giants and R is the AdS radius, and for large R (or r_1) we can get the string as long as

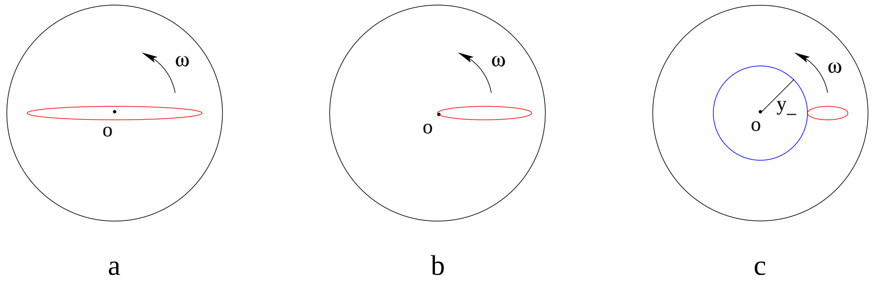

Figure 4: Semi-classical closed string solutions stretched along y direction with angular momentum in the "*AdS*" part and probing (a) AdS, (b) Superstar and (c) Giant graviton background.

we want and the background is exactly the AdS geometry given by a white hole in a black plane. From the previous section we note that this is also AdS solution. Our observation will also be supported in the next subsection by noting that for the region smaller than $\sqrt{r_1 r_2}$ the only possible solution will be a point like string which leads to the Penrose limit of the geometry that would be a plane-wave, in the LLM (x_1, x_2) plane notation is described by black in upper half plane and white in lower half plane.

3.2 Closed string probes rotating in S^3

Let us now consider a semi-classical closed string solution which is stretched along the y coordinate and rotates along $\theta \in S^3$ with speed of ω. The corresponding ansatz and conserved charges are given by

$$t = \kappa\tau\,, \qquad \theta = \omega\tau\,, \qquad y = y(\sigma) = y(\sigma + 2\pi) \tag{3.18}$$

$$E = \frac{\kappa}{2\pi\alpha'}\int_0^{2\pi} d\sigma\; 2y\cosh G\,, \qquad S = \frac{\omega}{2\pi\alpha'}\int_0^{2\pi} d\sigma\; y e^G\,. \tag{3.19}$$

In this case the Virasoro constraints read

$$\text{AdS}: y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{r_0^2} - \frac{\kappa^2}{y^2}\right) = 0 \tag{3.20a}$$

$$\text{Superstar}: y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{\rho r_0^2} - \frac{\kappa^2}{y^2 + (1-\rho)r_0^2}\right) = 0 \tag{3.20b}$$

$$\text{Giant}: y'^2 + y^2(y^2 + r_1^2)(y^2 + r_2^2)\left(\frac{\omega^2 - \kappa^2}{(r_1^2 - r_2^2)y^2} - \frac{\kappa^2}{y^4 + 2y^2 r_2^2 + r_1^2 r_2^2}\right) = 0\,. \tag{3.20c}$$

In the AdS case the periodicity condition can be satisfied for $\omega > \kappa$ and the turning point is given by $y_0^2 = \frac{\kappa^2}{\omega^2 - \kappa^2} r_0^2$. In the long string limit where $\omega \sim \kappa$ we get logarithmic correction to the energy, $E \sim S + \frac{r_0}{\pi\alpha'}\ln\frac{\alpha' S}{r_0}$, while in the short string limit where $\omega \gg \kappa$ we find the Regge trajectory as in the flat space $E^2 \sim r_0 \frac{2S}{\alpha'}$. (See figure 4$a$).

3.2.1 The case of superstar

The negative "potential" condition can only be satisfied when $\omega \geq \kappa$. The $\omega = \kappa$ case which only has a zero size string solution will be discussed later. For $\omega > \kappa$ the periodicity condition can be satisfied if $\frac{\kappa^2}{\omega^2} \geq 1-\rho$ in which the turning points are given by

$$y_0^2 = r_0^2 \left(\rho \frac{\omega^2}{\omega^2-\kappa^2} - 1 \right). \tag{3.21}$$

Note, however, that unlike the AdS case the string is not folded symmetrically around the origin. In fact we get folded orbiting string which starts from the origin and goes up to y_0 and then folds on itself. For the $y_0 = 0$ case, i.e. when $\frac{\kappa^2}{\omega^2} = 1-\rho$ (figure 4b), we will have a zero size string localized at the origin.

In the long string $\kappa \to \omega$ limit,

$$y_0^2 \sim \frac{\rho r_0^2}{\eta} \gg \rho r_0^2\,, \qquad \kappa \sim \frac{1}{2\pi} \ln \frac{\rho r_0^2}{\eta}\,, \qquad \omega \sim \frac{1}{2\pi}\sqrt{1+\eta}\ln\frac{\rho r_0^2}{\eta}\,, \tag{3.22}$$

where $\frac{\omega^2-\kappa^2}{\kappa^2} = \eta \to 0$. Using (3.19) in the leading order we arrive at

$$\begin{aligned} E &\sim \frac{r_0\sqrt{\rho}}{4\pi\alpha'}\left(\frac{1}{\eta} + \ln\frac{1}{\eta}\right) \\ S &\sim \frac{r_0\sqrt{\rho}}{4\pi\alpha'}\left(\frac{1}{\eta} - \ln\frac{1}{\eta}\right) \end{aligned} \tag{3.23}$$

or equivalently

$$E \sim S + \frac{r_0\sqrt{\rho}}{2\pi\alpha'} \ln \frac{4\pi\alpha' S}{r_0\sqrt{\rho}}\,. \tag{3.24}$$

In terms of N given in (3.5), $E - S \propto \sqrt{N/4\pi}\ln(4\pi S^2/N)$, which is the same as the AdS case except for the factor 2 in the second term that appears because in this case we are dealing with a closed string orbiting around the origin.

On the other hand in the short string limit, $\frac{\omega^2}{\omega^2-\kappa^2} = \frac{1}{\rho}+\xi$ with $\xi \to 0$, $y_0^2 = \xi\rho r_0^2 \ll \rho r_0^2$ and using (3.19) in the leading order we arrive at

$$\begin{aligned} E &\sim \frac{r_0}{\pi\alpha'}\,\frac{\kappa}{\sqrt{\omega^2-\kappa^2}}\,, \\ S &\sim \frac{(1-\rho)r_0}{\pi\alpha'}\,\frac{\omega}{\sqrt{\omega^2-\kappa^2}}\,. \end{aligned} \tag{3.25}$$

Therefore,

$$E^2 \sim -\frac{r_0^2}{\pi^2\alpha'^2} + \left(\frac{1}{1-\rho}\right)^2 S^2\,, \tag{3.26}$$

which is not the Regge trajectory in the flat space as expected. We note that there is a lower limit on the spin of the string, $S^2 \geq \left(\frac{1-\rho}{\rho}\right)^2 \frac{\rho r_0^2}{\pi^2\alpha'^2}$ or in terms $N, \mathcal{J}$:[9]

$$\frac{S^2}{N} \geq \frac{1}{\pi}\left(\frac{\mathcal{J}}{N^2}\right)^2 \tag{3.27}$$

[9]Here we have assumed $l_p^2 = \alpha'$.

and as expected in the $\rho \to 1(\mathcal{J} \to 0)$ limit the above lower bound on S goes to zero. Taking the above bound on S into account it is readily seen that

$$E^2 \geq \frac{4}{\pi} \frac{2\mathcal{J}}{N} \left(1 + \frac{2\mathcal{J}}{N^2}\right).$$

Moreover, there is no smooth $\rho \to 1$ limit which means that this state is not present in the AdS background. On the other hand for large S limit we find

$$E \approx \frac{1}{1-\rho} S - \frac{1}{2\pi^2 \alpha'^2} \frac{(1-\rho) r_0^2}{S}. \tag{3.28}$$

The above equation should be compared with (3.10). As we see in the leading order in S (3.28) is obtained from (3.10) by $\rho \to 1-\rho$, the $\mathbb{Z}_2$ transformation. This is in agreement with our earlier arguments about the $\mathbb{Z}_2$ which exchanges the S^3 and $\tilde{S}^3$. For integer values of $1/1-\rho$ (3.28) may correspond to an orbifold probed by closed strings (cf. discussions of section 3.1.1).

Eq. (3.28) has the same linear behavior as in the AdS in the leading term. One might then wonder if the small fluctuation of this string probes a plane wave geometry as well. We will return to this question in the next section.

It worth noting that in the superstar case we never get Regge trajectory which reflects the fact that the solution is singular. Actually we would expect to get flat Regge trajectory in the core of a solution if the effects of curvature is negligible. But in this case the geometry is singular exactly where we would expect to get flat Regge trajectory and therefore absence of the Regge trajectory signals the singularity of the background at the origin, at $x_1 = x_2 = y = 0$. We note also that, excluding the regime near the singularity, the other parts seen by rotating closed strings are essentially the same as the AdS background. In this sense the superstar geometry is closer to AdS backgraond then the giant graviton geometry.

3.2.2 The giant graviton case

In the giant graviton case the periodicity condition is not satisfied for $\omega < \kappa$. While for $\omega \geq \kappa$ one may have the closed string solution provided that

$$\frac{r_1}{r_2} \geq \frac{2\omega^2 - \kappa^2}{\kappa^2}, \tag{3.29}$$

in which the turning points of the closed string are given by

$$y_\pm^2 + r_2^2 = \frac{\kappa^2 (r_1^2 - r_2^2)}{2(\omega^2 - \kappa^2)} \left(1 \pm \sqrt{1 - \frac{4\omega^2 r_2^2 (\omega^2 - \kappa^2)}{\kappa^4 (r_1^2 - r_2^2)}}\right). \tag{3.30}$$

We note, however, that unlike the AdS case we will get folded orbiting closed string stretched along y direction for $y_- \leq y \leq y_+$ (see figure 4c). We also note that y_- changes from zero up to an upper limit given by $\sqrt{r_1 r_2}$ and reaches the bound, $y_- = \sqrt{r_1 r_2}$, for $\frac{r_1}{r_2} = \frac{2\omega^2 - \kappa^2}{\kappa^2}$ where we get zero size strings localized at $\sqrt{r_1 r_2}$. For $\omega = \kappa$ only zero size string satisfies the condition. The situation is as follows.

Let us start from the limit where $y_- = y_+ = \sqrt{r_1 r_2}$ in which the string has zero size, localized at $\sqrt{r_1 r_2}$ and with energy linearly proportional to its spin:

$$E = \frac{1}{2}\left(1 + \frac{r_1}{r_2}\right) S\,. \tag{3.31}$$

This happens when the equality in (3.29) has been satisfied. We note that the $r_2 \to 0$ is not a smooth limit and therefore this is a new sector that has occurred because of the presence of giant gravitons.

One may then change situation a little bit so that $y_\pm = \sqrt{r_1 r_2} \pm \epsilon$ where we would have a short closed string with length 2ϵ. For $\omega - \kappa \sim \mathcal{O}(1)$ we get closed string with the length of

$$l = \frac{\sqrt{\kappa^4(r_1^2 - r_2^2) - 4\omega^2 r_2^2(\omega^2 - \kappa^2)}}{(\omega^2 - \kappa^2)} \sqrt{\frac{r_1^2 - r_2^2}{r_1 r_2}}\,.$$

When κ approaches ω we will have long closed string and in the $\kappa \to \omega$ limit the string is stretched in y direction between the origin, $y_- \to 0$, and infinity $y_+ \to \infty$. This might be thought as the case when the periodicity condition is going to be lost and we are dealing with open string stretched all the way to infinity. Actually as we will discuss in section 3.4 the better description could be be given in terms of zero size string localized at the origin whose energy is linearly dependent on the spin at leading order, E=S, and the background observed by the fluctuations around this zero size classical solution would be the plane-wave solution.

3.3 Multi-spin string probes

In this section we briefly study the multi-spin closed string solutions in the backgrounds that we have been considering. In general since the LLM backgrounds have SO(4) $\times$ SO(4) $\times$ U(1)$_+$ isometry the most general solution one can consider is labeled by five quantum numbers, the energy and four quantum spin quantum numbers [27]. To be more precise let us consider the case with $r = 0$ and $\phi =$ constant where the Polyakov action reads

$$\begin{aligned} I = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \Big[& -h^{-2}\partial_\alpha t \partial^\alpha t + h^2 \partial_\alpha y \partial^\alpha y + \\ & + y e^{G}(\partial_\alpha \theta \partial^\alpha \theta + \sin^2\theta \partial_\alpha \psi_1 \partial^\alpha \psi_1 + \cos^2\theta \partial_\alpha \psi_2 \partial^\alpha \psi_2) + \\ & + y e^{-G}(\partial_\alpha \beta \partial^\alpha \beta + \sin^2\beta \partial_\alpha \gamma_1 \partial^\alpha \gamma_1 + \cos^2\beta \partial_\alpha \gamma_2 \partial^\alpha \gamma_2) \Big]\,. \end{aligned} \tag{3.32}$$

Note that we have changed the parametrization of two spheres to make the isometries manifest. For the isometry of the theory we have the following conserved charges

$$E = \frac{1}{2\pi\alpha'} \int d\sigma h^{-2} \partial_\tau t\,, \tag{3.33a}$$

$$S_1 = \frac{1}{2\pi\alpha'} \int d\sigma\; y e^{G} \sin^2\theta\; \partial_\tau \psi_1\,, \qquad S_2 = \frac{1}{2\pi\alpha'} \int d\sigma\; y e^{G} \cos^2\theta\; \partial_\tau \psi_2\,, \tag{3.33b}$$

$$J_1 = \frac{1}{2\pi\alpha'} \int d\sigma y e^{-G} \sin^2\beta\; \partial_\tau \gamma_1\,, \qquad J_2 = \frac{1}{2\pi\alpha'} \int d\sigma y e^{-G} \cos^2\beta\; \partial_\tau \gamma_2\,. \tag{3.33c}$$

3.3.1 The $S_2 = J_2 = 0$ case

As the first multi-spin example we consider the case in which the string rotates both in S^3 and $\tilde{S}^3$ as

$$t = \kappa\tau\,, \qquad \theta = \omega\tau\,, \qquad \psi = \nu\tau\,, \qquad y = y(\sigma) = y(\sigma + 2\pi)\,, \tag{3.34}$$

where the Virasoro constraints lead to

$$\text{AdS}: y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{r_0^2} + \frac{\nu^2 - \kappa^2}{y^2}\right) = 0 \tag{3.35a}$$

$$\text{Superstar}: y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{\rho r_0^2} + \frac{\nu^2 - \kappa^2}{y^2 + (1-\rho)r_0^2}\right) = 0 \tag{3.35b}$$

$$\text{Giant}: y'^2 + y^2(y^2 + r_1^2)(y^2 + r_2^2)\left(\frac{\omega^2 - \kappa^2}{(r_1^2 - r_2^2)y^2} + \frac{\nu^2 - \kappa^2}{y^4 + 2y^2 r_2^2 + r_1^2 r_2^2}\right) = 0\,. \tag{3.35c}$$

The general features of this solution is the same as what we have studied before. Indeed for $\nu < \kappa < \omega$ the physics is the same as the case the string rotates only in the "AdS" part (has only ω) while for $\omega < \kappa < \nu$ the string as if only $\nu \neq 0$ as we discussed above.

3.3.2 The $J_1 = J_2 = 0$ case

As the next example let us consider two-spin solution in "AdS" part of the metric. That is, we are looking for a solution describing a closed string rotating in both ψ_1 and ψ_2 where the ansatz would be

$$t = \kappa\tau\,, \quad \psi_1 = \omega_1\tau\,, \quad \psi_2 = \omega_2\tau\,, \quad y = y(\sigma) = y(\sigma+2\pi)\,, \quad \theta = \theta(\sigma) = \theta(\sigma+2\pi)\,. \tag{3.36}$$

The equations of motion and the Virasoro constraints are then given by

$$\begin{aligned}
&(h^2 y')' = \partial_y(h^2)y'^2 + \partial_y(h^{-2})\kappa^2 + \partial_y(ye^G)(\theta'^2 - \omega_1^2\sin^2\theta - \omega_2^2\cos^2\theta),\\
&(ye^G\theta')' = \frac{1}{2}(\omega_2^2 - \omega_1^2)ye^G\sin 2\theta\,,\\
&h^2 y'^2 + ye^G\theta'^2 = h^{-2}\kappa^2 - ye^G(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta)\,.
\end{aligned} \tag{3.37}$$

It is of course very difficult to solve these system of non-linear equations for generic values of parameters, though, it can be simplified by setting $\omega_1 = \omega_2 = \omega$ which implies

$$\theta' = \frac{c}{ye^G} \tag{3.38}$$

for a constant c. For zero c we recover the single spin case discussed in the previous sections, while for nonzero c the Virasoro constraints reads as

$$\text{AdS}: y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{r_0^2} + \frac{c^2/y^2 - \kappa^2}{y^2}\right) = 0 \tag{3.39a}$$

$$\text{Superstar}: y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{\rho r_0^2} + \frac{c^2/y^2 - \kappa^2}{y^2 + (1-\rho)r_0^2}\right) = 0 \tag{3.39b}$$

$$\text{Giant}: y'^2 + y^2(y^2 + r_1^2)(y^2 + r_2^2)\left(\frac{\omega^2 - \kappa^2}{(r_1^2 - r_2^2)y^2} + \frac{c^2/y^2 - \kappa^2}{y^4 + 2y^2 r_2^2 + r_1^2 r_2^2}\right) = 0\,. \tag{3.39c}$$

One can now proceed with the analysis of this equation to check whether the periodicity condition is satisfied and we get closed string solutions. We can then see how the string probe views different backgrounds. This multi-spin solution for AdS case has been studied in [27]. In the short string limit where the closed string is near the center of AdS we get usual Regge trajectory as in the flat space plus a correction due to the curvature of AdS. In the long string case where the string is close to the boundary of AdS one finds

$$E \approx 2S + \frac{3}{4}\frac{r_0^{2/3}}{\alpha'^{2/3}}\,(4S)^{1/3} + \mathcal{O}(S^{-1/3}) \tag{3.40}$$

which shows the first correction to $E - 2S$ goes as $S^{1/3}$ which is different from the logarithmic correction in the single spin case. Of course we note that this solution with large S is not stable [27].

In the remaining part of this subsection we will only briefly consider the giant graviton and superstar cases and postpone the detail to the future studies. Viewing (3.39c) as the zero energy condition with a given potential (cf. discussions of section 3.1), the periodicity condition can be satisfied only when the potential is negative or zero and has a negative slope in the same region. This can be achieved when

$$4(\omega^2 r_2^2 + c^2)(\omega^2 - \kappa^2) \le \kappa^4 (r_1^2 - r_2^2)\,, \tag{3.41}$$

in which the turning points are given by

$$y_\pm^2 + r_2^2 = \frac{\kappa^2(r_1^2 - r_2^2)}{2(\omega^2 - \kappa^2)}\left(1 \pm \sqrt{1 - \frac{4(\omega^2 r_2^2 + c^2)(\omega^2 - \kappa^2)}{\kappa^2(r_1^2 - r_2^2)}}\right). \tag{3.42}$$

Here we just consider the simplest example where $y = y_0$ is constant which means that the string becomes circular and is stretched only in θ direction. For this situation we get

$$\theta'^2 = f_1\kappa^2 - \omega^2\,, \qquad \theta'^2 = -f_2\kappa^2 + \omega^2\,, \tag{3.43}$$

where

$$f_1 = \frac{(y_0^2 + r_1^2)(y_0^2 + r_2^2)}{y_0^4 + 2y_0^2 r_2^2 + r_1^2 r_2^2}\,, \qquad f_2 = \frac{y_0^2(y_0^2 + r_2^2)^2 + r_2^2(y_0^2 + r_1^2)^2}{(y_0^2 + r_2^2)(y_0^4 + 2y_0^2 r_2^2 + r_1^2 r_2^2)}\,. \tag{3.44}$$

One can easily solve this equation which gives $\theta = w\sigma$ where w is the winding number of the string around θ. For the $w = 1$ case we get

$$\kappa^2 = 2\frac{(y_0^2 + r_2^2)(y_0^4 + 2y_0^2 r_2^2 + r_1^2 r_2^2)}{(y_0^4 - r_2^2 r_1^2)(r_1^2 - r_2^2)}\,, \qquad \omega^2 = \frac{(2y_0^2 + r_2^2 + r_1^2)(y_0^4 + 2y_0^2 r_2^2 + r_1^2 r_2^2)}{(y_0^4 - r_2^2 r_1^2)(r_1^2 - r_2^2)}\,. \tag{3.45}$$

The conserved charges for this solution are given by

$$E = \frac{\kappa}{\alpha'} h_0^{-2}\,, \qquad S_1 = S_2 = S = \frac{\omega}{2\alpha'}(ye^G)_0\,. \tag{3.46}$$

Note that both E and S are functions of ω and κ and y_0. By making use of (3.45) one may find $E = E(y_0)$ and $S = S(y_0)$ as follows

$$E = \frac{\sqrt{2}}{\alpha'}\,\frac{(y_0^2 + r_1^2)(y_0^2 + r_2^2)^{3/2}}{(y_0^4 - r_2^2 r_1^2)^{1/2}(r_1^2 - r_2^2)}\,, \qquad S = \frac{1}{2\alpha'}\frac{(2y_0^2 + r_2^2 + r_1^2)^{1/2}(y_0^4 + 2y_0^2 r_2^2 + r_1^2 r_2^2)}{(y_0^4 - r_2^2 r_1^2)^{1/2}(r_1^2 - r_2^2)}\,. \tag{3.47}$$

One can now eliminate y_0 from these expression to find the dependence of energy on the spin. For example in the large y_0 limit one gets

$$E \approx 2S + \frac{3}{4}\,\frac{(r_1^2 - r_2^2)^{1/3}}{\alpha'^{2/3}}(4S)^{1/3} + \mathcal{O}(S^{-1/3}) \tag{3.48}$$

which has the same form as in the AdS case [27], namely the first correction to $E - 2S$ goes as $S^{1/3}$ unlike the single spin where we had logarithmic correction.

On the other hand unlike the AdS case we cannot get small y_0 limit and therefore we wont get Regge trajectory for short string limit [27]. In fact from the Virasoro constraint we observe that in order to get a well-behaved solution one needs to have $y_0^2 = \frac{2\omega^2-\kappa^2}{\kappa^2}r_2^2 + \frac{2c^2}{\kappa^2}$ or

$$y_0^4 = r_1^2 r_2^2 + \frac{4c^2}{\kappa^4}(c^2 + \omega^2 r_2^2)\,. \tag{3.49}$$

Therefore, the shortest string one can have is of order of $\sqrt{r_1 r_2}$ which is given in the limit of $c \to 0$ where we get the single spin solution. As a conclusion the circular multi-spin closed strings only exist for radius bigger that $\sqrt{r_1 r_2}$ where we get the same behavior as in the AdS case.

One can also do the same computations for superstar case. For the superstar case in the long string limit we get exactly the same result as AdS case i.e.

$$E \approx 2S + \frac{3}{4}\frac{\rho^{1/3} r_0^{2/3}}{\alpha'^{2/3}}\,(4S)^{1/3} + \mathcal{O}(S^{-1/3})\,. \tag{3.50}$$

On the other hand in the short string case we will get

$$E \approx \left(\frac{1}{1-\rho}\right)^{1/2} S \tag{3.51}$$

which is not the Regge trajectory and in fact is very similar to the single spin case where we get linear behavior. This again confirms our observation that apart the near core limit the superstar solution behaves very similar to AdS and the effect of the singularity changes the behavior of the string near the core where unlike the AdS case we won't get Regge trajectory of flat space. Instead we get a linear behavior which has no smooth $\rho \to 1$ limit reflecting the fact that the geometry is singular and the string feels the singularity.

3.4 The plane-wave limits

As we have seen in the previous subsections there are cases where the string gets zero size. We have also studied the dependence of energy on spin in the leading order. Of course the better treatment would be to study small fluctuations around these point like string solutions. For example in the AdS background for the closed string which rotates only in S^5 part the periodicity condition can only be satisfied if the string is shrunk to zero size, localized at the origin while rotating with the speed of light. The fluctuation around this classical solution leads to the plane-wave solution of the AdS background [3, 4]. From LLM point of view this can be done by focusing on a small region around the edge of the disk in the (x_1, x_2) plane and then blowing up this region. The boundary condition we get corresponds to the plane-wave solution (see figure 5a). Practically the procedure can be

Figure 5: The boundary conditions corresponding to the plane-wave limit of AdS (a) and superstar (b) solutions.

done by taking the limit of $r_0 \to \infty$ and keeping x_1, x_2 and w fixed, where

$$r - r_0 = \frac{x_2}{r_0}\,, \qquad y = \frac{w}{r_0}\,, \qquad \phi - \frac{\pi}{2} = \frac{x_1}{r_0^2} \tag{3.52}$$

which leads to

$$z = \frac{x_2}{2\sqrt{x_2^2 + w^2}}\,, \qquad V_1 = -V_\phi \partial_1 \phi = \frac{1}{2\sqrt{x_2^2 + w^2}}\,, \qquad V_2 = 0\,. \tag{3.53}$$

Having had the zero size string in the superstar and giant graviton cases, one might be wondering whether the same physics can appear there. It is very messy to study small fluctuations around this classical solutions for these cases, nonetheless one may follow the above procedure to find the plane-wave limit in these cases as well.

In the superstar case we can again focus on a region near the edge of the gray disk and then blow up the region. We note, however, that there are two different ways to do that. If we focus on the region and then blow it up by taking $r_0 \to \infty$ and $\rho \to 1$ limit while keeping $(1 - \rho)r_0$ fixed one will get the boundary condition as in figure 5a, namely the geometry we obtain is the plane-wave limit of AdS solution. On the other hand one may consider the case where r_0 goes to infinity while keeping ρ fixed. More precisely

$$r - r_0 = \frac{x_2}{r_0}\,, \qquad y = \frac{w}{r_0}\,, \qquad \phi - \frac{\pi}{2} = \frac{x_1}{r_0^2}\,. \tag{3.54}$$

In this limit one gets

$$\tilde{z} = \frac{\rho}{2}\left(\frac{x_2}{\sqrt{x_2^2 + w^2}} - 1\right), \qquad V_1 = \frac{\rho}{2}\,\frac{1}{\sqrt{x_2^2 + w^2}}\,, \qquad V_2 = 0\,, \tag{3.55}$$

which corresponds to a singular plane-wave solution given by the boundary condition as in figure 5b. Similarly to the superstar solution this Penrose limit heads to a space with null, naked singularity.

In sum, there are two ways of taking the Penrose limit of half BPS superstar solution which has a naked singularity. One leads to the smooth maximally supersymmetric plane-wave geometry and the other to a 1/2 BPS solution with naked singularity. In this regards it is quite similarly to the Penrose limits of $AdS_5 \times S^5/Z_k$ orbifolds [26] (recall also the discussions of section 3.1.1).

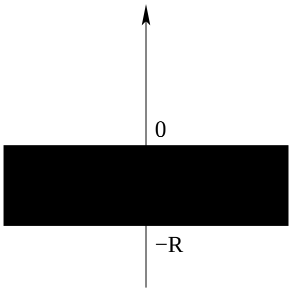

Figure 6: Penrose limit of deformed AdS by giant gravitons.

In the case of the giant graviton similarly there are two Penrose limits. In the first one we blow up the edge of the droplet by taking large r_1 limit while keeping r_2 fixed. This leads to the plane-wave limit of AdS, as if there are no giant gravitons. There is also another limit one may consider, namely [25, 28]

$$y = \frac{w}{r_1}\,, \qquad r - r_1 = \frac{x_2}{r_1}\,, \qquad r_1 - r_2 = \frac{R}{r_1}\,, \qquad \phi - \frac{\pi}{2} = \frac{x_1}{r_1^2} \tag{3.56}$$

which in the limit of $r_1 \to \infty$ keeping w, x_1, x_2 and R fixed, we get

$$\begin{aligned} \tilde{z} &= \frac{x_2}{2\sqrt{x_2^2 + w^2}} - \frac{x_2 + R}{2\sqrt{(x_2+R)^2 + w^2}}\,, \qquad V_2 = 0\,, \\ V_1 &= \frac{1}{2\sqrt{x_2^2 + w^2}} - \frac{1}{2\sqrt{(x_2+R)^2 + w^2}}\,. \end{aligned} \tag{3.57}$$

In this case we send the size of the giant gravitons to infinity in the same rate as r_1 or r_2 and hence after the limit the spherical brane becomes a flat three brane. For this solution the boundary condition is given by a long strip in the (x_1, x_2) plane as figure 6.

4. Discussions and conclusions

In this paper we compared two half BPS deformations of $AdS_5 \times S^5$, one is LLM type which is a smooth deformation of $AdS_5 \times S^5$ with some number of giant gravitons on the S^5 and the other is of the form of superstar with naked singularity. We chose the two backgrounds to have the same number of the fiveform flux over the S^5, N, and with the same angular momentum along an $S^1 \in S^5$, $\mathcal{J}$ and used closed string probes. As we showed closed strings probes can distinguish the two backgrounds and the fact that the superstar geometry is singular. In particular we did not find Regge trajectory behavior for strings in the superstar case, showing that the string probe feels a region with large curvature, rather than an approximately flat space, a sign of naked curvature singularity.

In section 2 of this paper we introduced and discussed a $\mathbb{Z}_2$ symmetry of the LLM backgrounds, which can also be extended to the superstar geometries. As we discussed the closed string probes also show the same $\mathbb{Z}_2$ symmetry.

One of the intriguing points discussed was the relation between energy and the angular momentum and/or spin of the closed string probe for the superstar case, (3.10), (3.28). Of course these relations do not have any counterpart in the LLM geometry of giant gravitons.

JHEP11(2005)005

As discussed for integer values of $1/\rho$ or $1/1-\rho$ this relations are exactly those one would expect from a short string probing $AdS_5 \times S^5/Z_k$ or $AdS_5/Z_k \times S^5$ 1/2 BPS orbifolds, where $1/\rho$ and $1/1-\rho$ are equal to k. In our case, however, we did not have any argument as to why $1/\rho$ should be an integer. This condition may arise as a consistency condition once we quantize our semi-classical closed string probes. We also mentioned a possible relation to fractional quantum Hall effect. As we discussed the superstar solution can be obtained as a "classical" limit of a smooth LLM geometry (cf. footnote 7). One may then wonder whether the converse if also true, i.e. whether a smooth LLM type geometry can be obtained from a collection of superstars. Recalling the superposition property of the LLM type solutions, this may seem possible for integer values of $1/\rho$. For example one may expect to obtain AdS geometry, a black disk, from $1/\rho$ number of superstars, gray disks, sitting on top of each other. Clarifying this relation and the quantization of ρ (which in the language of the corresponding quantum Hall system is nothing but the density of fermions or the filling factor) is a question with an obvious interest. This is postponed to feature works.

In this paper we have only considered the closed string solution, though one could also study open strings as well. In this case we should drop the periodicity condition in σ and instead consider an open string which would have been string stretched all the way to boundary if we had been in the pure AdS. But in our case one could interpret them as open strings ending on the giant gravitons. Actually one might imagine then as closed strings opened up and attached on giant gravitons.

As we have seen in the giant graviton case the closed string with angular momentum in the "AdS" part it is not possible to be folded symmetrically around the origin and we could only find an orbiting folded string. The situation is very similar to the case when we have horizon, like the case of AdS-Schwarzchild blackhole. We note, however, that our background solutions have no horizon. As we see the folded closed strings can easily recognize the giant graviton background from the pure AdS solution. In the AdS case one only has closed string folded symmetrically around the origin, whereas in the giant case the string orbits around the origin. One may think that adding giant gravitons will split the closed string into two parts (or open them up with the ends on the giants) and therefore the limit going from giant graviton to pure AdS is not smooth. In particular in the giant graviton case there is string localized at $\sqrt{r_1 r_2}$ which does not have counterpart in the pure AdS case. In fact as we see from (3.31) the $r_2 \to 0$ limit is not a smooth one. From closed strings point of view, it does not matter how small r_2 is, as soon as it is nonzero the string probes a new physics. This might be interpreted as the fact how the string could probe the quantum nature of the (x_1, x_2) plane.

As has been discussed in the literature [21, 22] the superstar case with $\rho > 1$ corresponds to a geometry with closed time-like curves, a background with naked timelike singularity. It is interesting to briefly analyze this case from the closed string probes view-point. Let us again recall (3.35b):

$$y'^2 + y^2(y^2 + r_0^2)\left(\frac{\omega^2 - \kappa^2}{\rho r_0^2} + \frac{\nu^2 - \kappa^2}{y^2 + (1-\rho)r_0^2}\right) = 0\,.$$

For $\rho > 1$ there are poles at $y = \pm\sqrt{(\rho-1)}r_0$ which changes the situation drastically. Further study of this case which can shed further light on the nature of these closed time-like curves is postponed to future works.

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